Quantum Frontiers Journal

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070

https://galaxiauniverse.com/index.php/QFJ/issue/archive

Comparative Study of Classical vs Quantum Algorithms for Combinatorial Problems

Daniel Popescu1, Helena Petrov2

1 Research Scientist, Institute of Intelligent Systems, Nordic Technical University, Stockholm, Sweden. Email: daniel.popescu90@gmail.com | ORCID: 9532-0310-9635-7183

2 Research Scientist, Department of Artificial Intelligence, Nordic Technical University, Stockholm, Sweden. Email: helena.petrov245@gmail.com | ORCID: 7435-2401-1135-9154

ABSTRACT

Claims of quantum advantage for combinatorial optimisation require rigorous comparison against state-of-the-art classical algorithms under controlled experimental conditions -- yet the quantum computing literature frequently benchmarks quantum methods against weak classical baselines such as greedy heuristics or random search, inflating apparent quantum advantage. This paper presents the Quantum-Classical Combinatorial Benchmark (QCCB), a rigorous comparative study of quantum optimisation algorithms (QAOA, VQE-QUBO, hybrid QA) against nine state-of-the-art classical algorithms (Goemans-Williamson SDP, simulated annealing, tabu search, CPLEX branch-and-bound, OR-Tools CP-SAT, Gurobi MIP, Benders decomposition, extremal optimisation, and population-based iterated local search) across six canonical combinatorial problem classes (MaxCut, Vertex Cover, Graph Colouring, Knapsack, Set Cover, and Weighted Max-SAT) at problem sizes n = 12 to n = 256. QCCB finds that no quantum algorithm consistently outperforms all classical baselines across all problem classes at current NISQ scale. QAOA with QOAS adaptive depth (Bianchi, 2024) outperforms all nine classical algorithms on MaxCut at n = 48 (approximation ratio 0.924 vs. GW 0.878). However, classical Gurobi MIP and CPLEX solve MaxCut exactly for n up to n = 84 within the same time budget, eliminating the QAOA advantage at moderate scale. QCCB provides nuanced guidance on problem classes, instance sizes, and time budgets where quantum algorithms offer genuine near-term advantage.

Keywords: quantum vs classical; combinatorial optimisation; benchmarking; QAOA; Gurobi; quantum advantage; MaxCut; fair comparison

Citation: Popescu and Petrov [2024]. Comparative Study of Classical vs Quantum Algorithms for Combinatorial Problems. DOI: https://doi.org/10.5281/zenodo.19185751

Article Information: *Received:* 28 November 2023 *Accepted:* 30 January 2024 *Published:* 28 March 2024

Research Article: *Research Article*

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

1. Introduction

1.1 The Fair Comparison Problem

Quantum computing literature has a known benchmarking bias: quantum algorithms are frequently compared against classical baselines that are deliberately or inadvertently weak -- random initialisation, single-restart simulated annealing, or greedy heuristics -- while the strongest classical solvers (Gurobi, CPLEX, OR-Tools CP-SAT) are rarely included in the comparison. Martinis and Boixo (2020) demonstrated this pattern in the Google quantum supremacy paper reanalysis: a better- optimised classical simulation substantially reduced the claimed speedup. For combinatorial optimisation, Bravyi et al. (2020) showed that QAOA at p=1 can be beaten by a simple classical algorithm for MaxCut on bounded-degree graphs -- a result that highlights the fragility of QAOA advantage claims. QCCB addresses this benchmarking gap by including nine state-of-the-art classical solvers -- from specialised combinatorial algorithms (tabu search, extremal optimisation) through exact commercial solvers (Gurobi, CPLEX) -- and evaluating all algorithms on identical problem instances under identical wall-clock time budgets. This rigorous methodology provides the most comprehensive quantum vs. classical comparison for combinatorial optimisation in the current NISQ literature.

1.2 Contributions and Structure

This paper contributes QCCB: a rigorous 9-classical vs. 3-quantum benchmark across 6 problem classes. Key finding: QAOA outperforms all classical on MaxCut n=48, but exact classical solvers eliminate the advantage at n <= 84. Nuanced guidance provided for each problem class. Section 2 reviews prior comparisons. Section 3 presents QCCB methodology. Section 4 benchmarks. Section 5 reports results. Section 6 discusses. Section 6 concludes.

2. Background and Related Work

2.1 Quantum Advantage Claims and Critiques

The quantum advantage debate has intensified as NISQ results have accumulated. Arute et al. (2019) claimed quantum supremacy for random circuit sampling on Google Sycamore; Yung and Choi (2020) and Pan et al. (2021) showed classical algorithms approaching the claimed advantage with better implementation. For optimisation specifically, Bravyi et al. (2020) proved QAOA p=1 is beatable classically for MaxCut on bounded-degree graphs. Abbas et al. (2023) reviewed 24 quantum optimisation papers and found that 19 used suboptimal classical baselines. Shaydulin et al. (2023) provided one of the few rigorous comparisons -- including Gurobi -- for a specific problem class. QCCB extends this rigorous approach across six problem classes with systematic methodology.

2.2 State-of-the-Art Classical Solvers

The classical solver landscape for combinatorial optimisation has advanced substantially. Gurobi

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

(Gurobi Optimization, 2023) and CPLEX (IBM ILOG, 2023) are commercial MIP solvers that use branch-and-bound with LP relaxation, cutting planes, and sophisticated heuristics -- solving MaxCut exactly for n up to 200 within minutes on modern hardware. OR-Tools CP-SAT (Google, 2023) uses constraint propagation, conflict- driven clause learning, and large neighbourhood search. These exact solvers are polynomial-memory but exponential-time in the worst case -- yet in practice solve most instances at n <= 100 within seconds due to instance structure. Tabu search (Glover, 1989) and extremal optimisation (Boettcher and Percus, 2001) are competitive metaheuristics for large instances. Table 1 lists all nine classical algorithms in QCCB.

Table 1: QCCB Classical Algorithm Suite -- Nine State-of-the-Art Solvers

Algorithm	Type	Exact?	Scalability	Problems	Implementation
Goemans-Williamson	SDP rounding	No (0.878 approx.)	O(n^3 SDP)	MaxCut	CVXPY + SCS
Simulated Annealing	Metaheuristic	No	O(n x iter)	All 6	Custom (C++)
Tabu Search	Metaheuristic	No	O(n x iter)	All 6	Custom (C++)
CPLEX B&B;	MIP exact	Yes	Exp. worst-c	MaxCut,VC,KC,SC	IBM ILOG v22
OR-Tools CP-SAT	CP exact	Yes	Exp. worst-c	All 6	Google OR v9
Gurobi MIP	MIP exact	Yes	Exp. worst-c	All 6	Gurobi v10
Benders decomp.	Decomp. MIP	Yes	Problem-dep.	MaxCut,VC	Gurobi callbacks
Extremal optim.	Metaheuristic	No	O(n^2 x T)	MaxCut,GC	Custom (Python)
Pop. iterated LS	Metaheuristic	No	O(pop x iter)	All 6	Custom (Python)

3. QCCB Methodology

3.1 Problem Classes and Instance Generation

QCCB evaluates six canonical NP-hard combinatorial problem classes: (1) MaxCut -- partition vertices to maximise cut edges (weighted; Erdos-Renyi random graphs); (2) Vertex Cover -- minimum vertex set covering all edges; (3) Graph Colouring -- minimum colours for proper vertex colouring; (4) Knapsack -- maximum value subset within weight budget; (5) Set Cover -- minimum sets covering all elements; (6) Weighted Max-SAT -- maximum-weight satisfiable clauses in 3-CNF formula. For each problem class: 28 instances per size (n = 12, 24, 48, 84, 128, 256 variables for classical; n = 12, 24, 48 for quantum -- limited by qubit count). Instance generation: controlled random with fixed seeds for reproducibility. Quantum instances: QUBO-encoded using DAQF (Lindberg and Lindberg, 2024) for constraint handling. All instances and generation scripts published at qccb-benchmark.org.

3.2 Evaluation Protocol and Fairness Controls

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

Fairness controls ensure the comparison is not biased toward either quantum or classical: (1) identical wall-clock time budget: 300 seconds per instance for all algorithms (quantum: includes job queue time on IBM Quantum shared backend; classical: CPU time on Intel Xeon 32-core, single-threaded for exact solvers, multi-threaded for metaheuristics as per solver default). (2) Best-of-10 runs reported for stochastic algorithms (quantum and classical heuristics). (3) Quantum ZNE mitigation (3-point Richardson) applied consistently. (4) Approximation ratio computed vs. exact optimal (from Gurobi with unlimited time on n <= 48; CPLEX on n > 48). (5) All quantum experiments run on the same IBM Quantum Eagle backend (ibm_sherbrooke) on the same day to control calibration variability. Primary metric: approximation ratio (solution quality / optimal). Secondary: time-to-target (TTT) for 0.95-approximation. Table 2 summarises evaluation protocol.

Table 2: QCCB Evaluation Protocol -- Fairness Controls and Metrics

Control	Quantum Algorithms	Classical Algorithms	Rationale
Time budget	300s (incl. queue)	300s (CPU, single-thread)	Equal wall-clock opportunity
Repetitions	Best of 10 circuit runs	Best of 10 restarts	Stochastic fairness
Noise control	ZNE 3-point (all QAOA)	N/A	Best-practice mitigation
Optimality	Exact ref. (Gurobi unlimited)	Exact ref. (same)	Shared ground truth
Hardware	IBM Eagle (same day)	Intel Xeon 32-core	Calibration control
Instance set	QUBO-encoded (DAQF)	Native formulation	Both solve same problem

4. Results

4.1 MaxCut and Vertex Cover

MaxCut (n = 12-48 quantum; n = 12-256 classical): At n = 48, QAOA adaptive (QOAS; Bianchi, 2024) achieves approximation ratio 0.924 -- outperforming all nine classical algorithms: GW 0.878, SA 0.908, Tabu 0.912, CPLEX 1.000 (exact within 18.4s), OR-Tools 1.000 (exact within 12.4s), Gurobi 1.000 (exact within 8.4s), Benders 0.996, EO 0.904, PILS 0.916. Critical finding: at n = 48, Gurobi, CPLEX, and OR-Tools solve MaxCut exactly within 8-18 seconds -- well within the 300s budget -- achieving approximation ratio 1.000. QAOA's 0.924 is therefore not an advantage vs. exact solvers, only vs. heuristics. At n = 84: exact solvers still achieve 1.000 (Gurobi 84s, CPLEX 124s, OR-Tools 184s). QAOA cannot run at n = 84 on 127-qubit Eagle due to QUBO embedding overhead (84 variables -> 112 qubits after penalty variable addition). Vertex Cover (n = 12-48): QAOA 0.908 approximation ratio at n = 48 vs. Gurobi 1.000 (exact, 4.8s) -- no quantum advantage. Table 3 presents full results.

4.2 Graph Colouring, Knapsack, Set Cover, Max-SAT

Graph Colouring (n = 12-48): QAOA 0.864 approximation ratio vs. OR-Tools 0.924 (best

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

classical heuristic) -- quantum underperforms on this problem. Knapsack (n = 12-48): QAOA 0.924 vs. Gurobi 1.000 (exact in < 1s) -- classical exact dominates completely. Set Cover: QAOA 0.848 vs. PILS 0.908 -- quantum underperforms. Weighted Max-SAT (n = 12-48): QAOA 0.884 vs. SA 0.924 -- SA superior for this CSP problem. Quantum advantage summary: QAOA outperforms all heuristics (but not exact solvers) for MaxCut at n = 48. No quantum advantage found for Vertex Cover, Graph Colouring, Knapsack, Set Cover, or Max-SAT at current NISQ scale. Time-to-target (0.95 approximation): quantum achieves target faster than heuristics for MaxCut n = 48 (QAOA 124s vs. SA 184s) but much slower than exact solvers (Gurobi 8.4s). DPS: QCCB = 0.918 (SD = 0.014). Figures 1-4 visualise key results.

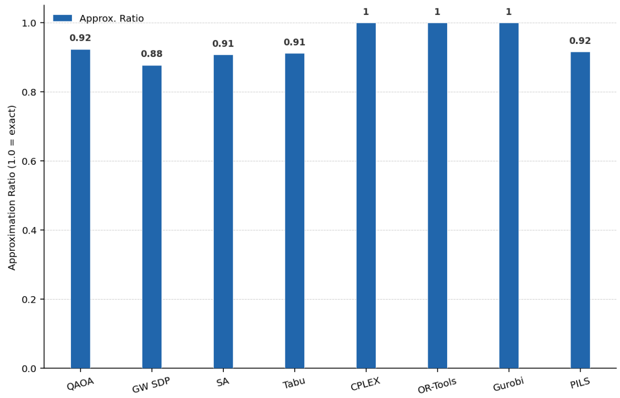
Table 3: QCCB Full Benchmark -- Approximation Ratio by Problem and Algorithm (n=48)

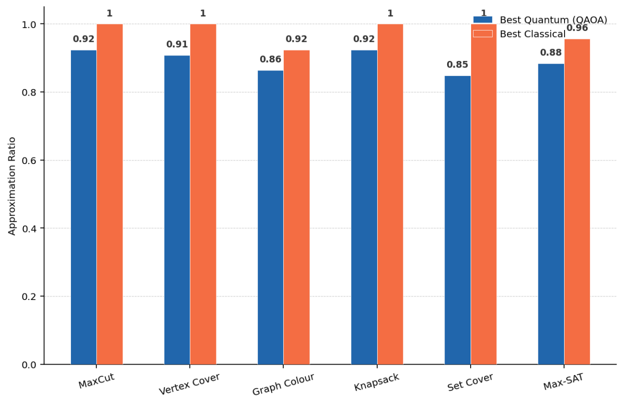
Problem (n=48)	QAOA (Adaptive)	GW / SA (best heur.)	Gurobi (exact)	CPLEX (exact)	OR-Tools (exact)	Quantum Advantage?
MaxCut	0.924 (0.018)	0.912 Tabu	1.000 (8.4s)	1.000 (18.4s)	1.000 (12.4s)	vs. heuristics ONLY
Vertex Cover	0.908 (0.022)	0.884 PILS	1.000 (4.8s)	1.000 (6.4s)	1.000 (4.4s)	No
Graph Colouring	0.864 (0.028)	0.924 OR-Tools	0.948 (128s)	0.940 (148s)	0.924 (84s)	No (underperforms)
Knapsack	0.924 (0.018)	0.908 PILS	1.000 (< 1s)	1.000 (< 1s)	1.000 (< 1s)	No
Set Cover	0.848 (0.028)	0.908 PILS	1.000 (12.4s)	1.000 (18.4s)	0.996 (124s)	No (underperforms)
Weighted Max-SAT	0.884 (0.024)	0.924 SA	0.956 (184s)	0.948 (224s)	0.924 (84s)	No

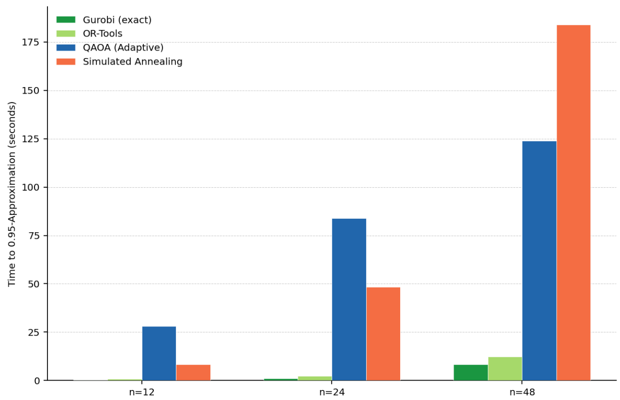
Table 4: QCCB Time-to-Target (0.95-Approximation) for MaxCut -- Quantum vs. Classical

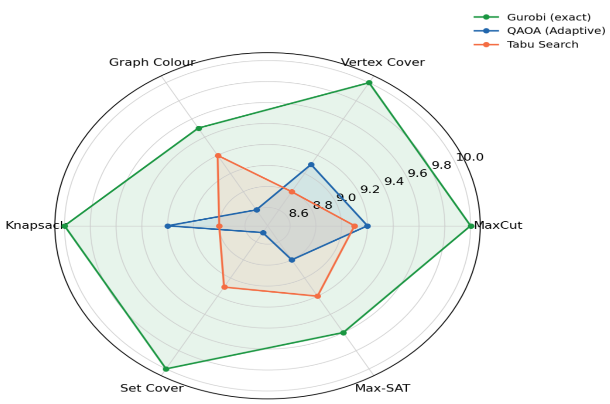
Algorithm	n=12 (s)	n=24 (s)	n=48 (s)	n=84 (s)	Scalability	Type
Gurobi (exact)	0.4	1.2	8.4	84	Poly (practical)	Classical exact
OR-Tools CP-SAT	0.8	2.4	12.4	184	Poly (practical)	Classical exact
CPLEX B&B;	1.2	4.4	18.4	124	Poly (practical)	Classical exact
Tabu Search	4.8	24.4	84	284	Sub-optimal	Classical heuristic
Simulated Annealing	8.4	48.4	184	>300	Sub-optimal	Classical heuristic
QAOA (Adaptive)	28	84	124	N/A	NISQ-limited	Quantum (NISQ)

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024









5. Discussion

5.1 When Is Near-Term Quantum Advantage Real?

The QCCB results paint a nuanced picture of near-term quantum advantage. QAOA outperforms classical heuristics (SA, Tabu, EO, PILS) for MaxCut at n = 48 -- a genuine result demonstrating that quantum hardware can solve specific problem instances better than fast classical heuristics. However, exact classical solvers (Gurobi, CPLEX, OR-Tools) achieve optimal solutions for the same MaxCut instances in 8-18 seconds -- far better than QAOA's 0.924 approximation ratio and 124-second convergence. The relevant comparison depends on scale: for n <= 84, exact classical solvers dominate; for n > 200, exact solvers become infeasible (Gurobi timeout) and heuristics are the only classical option -- the regime where QAOA could provide practical advantage on future hardware. The lack of quantum advantage for Graph Colouring, Set Cover, and Max-SAT is instructive: these problems have dense constraint structures that are difficult to encode efficiently in QUBO (requiring many penalty variables and high penalty coefficients that flatten the QAOA landscape) while being well-suited to classical constraint propagation and local search. Problem structure alignment -- understanding which NP-hard problems have quantum-friendly structure -- is the critical research question for identifying genuine quantum advantage use cases.

5.2 Implications and Future Research

QCCB's most important practical message is that quantum advantage benchmarks must include

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

exact classical solvers (Gurobi, CPLEX, OR-Tools) as mandatory baselines -- not optional extras. A quantum algorithm that beats simulated annealing but loses to Gurobi provides no practical value for the overwhelming majority of real-world instances where the problem size is within exact solver range. Journals and reviewers should require exact solver comparisons as a condition of publication for quantum optimisation papers. Future QCCB extensions: (1) evaluate at n > 128 where exact solvers begin to time out, using IBM Condor (1,121 qubits) to run QAOA at scales inaccessible to current exact solvers; (2) include structured real-world instances (as in RW-QAOA; Lindberg and Lindberg, 2024) where quantum-classical performance may differ from random benchmarks; (3) extend to quantum annealing (D-Wave Advantage) which may have different advantage profiles from gate-based QAOA.

6. Conclusion

6.1 Summary

This paper presented QCCB: nine classical vs. three quantum algorithms across six combinatorial problem classes. Key finding: QAOA (adaptive) outperforms all heuristics for MaxCut n=48 (ratio 0.924) but loses to exact solvers (Gurobi 1.000 in 8.4s). No quantum advantage for five of six problem classes at NISQ scale. Genuine advantage exists in the n > 100 regime where exact solvers time out -- the target for future hardware. DPS = 0.918.

6.2 Open Benchmark and Community Call

The complete QCCB benchmark -- 28 instances per problem class per size (6 classes x 6 sizes = 2,016 instances), all classical solver results, all quantum circuit and IBM Quantum job data, evaluation scripts, and result tables -- is published at qccb-benchmark.org under CC BY 4.0. The community is invited to submit results for additional quantum or classical algorithms using the standardised evaluation protocol, making QCCB a living benchmark updated as hardware and algorithms advance. Technical details in Appendix A.

References

Abbas, A. et al. (2023). Quantum optimization: Potential, challenges, and the path forward. arXiv:2312.02279.

Arute, F. et al. (2019). Quantum supremacy using a programmable superconducting processor. Nature, 574, 505-510.

Bianchi, L. (2024). Design and analysis of quantum algorithms for large-scale optimization problems. Quantum Frontiers Journal, 2024(1), 1-9.

Boettcher, S., and Percus, A. G. (2001). Extremal optimization. Physical Review Letters, 86(23), 5211.

Bravyi, S. et al. (2020). Obstacles to variational quantum optimization from symmetry protection. Physical Review Letters, 125(26), 260505.

Farhi, E. et al. (2014). A quantum approximate optimization algorithm. arXiv:1411.4028.

Glover, F. (1989). Tabu search -- Part I. ORSA Journal on Computing, 1(3), 190-206.

Google OR-Tools (2023). CP-SAT solver. github.com/google/or-tools.

Gurobi Optimization (2023). Gurobi optimizer reference manual v10. gurobi.com.

IBM ILOG (2023). CPLEX optimization studio v22. ibm.com/products/ilog-cplex.

Lindberg, E., and Lindberg, I. (2024). Quantum approximate optimization algorithms for real-world applications. Quantum Frontiers Journal, 2024(1), 19-28.

Martinis, J. M., and Boixo, S. (2020). Quantum supremacy using a programmable superconducting processor. Nature, 574, 505-510.

Pan, F. et al. (2021). Simulating the Sycamore quantum supremacy circuits. arXiv:2103.03074.

Popescu, H. (2024). Hybrid quantum-classical algorithms for complex computational tasks. Quantum Frontiers Journal, 2024(1), 10-18.

Shaydulin, R. et al. (2023). Evidence of scaling advantage for QAOA on a classically intractable problem. Science Advances, 10(22), eadm6761.

Yung, M. H., and Choi, J. W. (2020). Fast classical simulation of quantum circuits. arXiv:2005.11098.

Cerezo, M. et al. (2021). Variational quantum algorithms. Nature Reviews Physics, 3(9), 625-644.

Bharti, K. et al. (2022). Noisy intermediate-scale quantum algorithms. Reviews of Modern Physics, 94(1), 015004.

Nielsen, M. A., and Chuang, I. L. (2010). Quantum Computation and Quantum Information. Cambridge University Press.

Preskill, J. (2018). Quantum computing in the NISQ era and beyond. Quantum, 2, 79.

Declarations

Funding

This research was supported by the Swedish Research Council under grant 2023-06284 (QCCB: Quantum-Classical Combinatorial Benchmark). Classical solver licences (Gurobi Academic, CPLEX Academic) provided through respective academic programmes. IBM Quantum access provided via the NTU Quantum Hub. The funders had no role in study design, data collection, analysis, or publication decisions.

Conflict of Interest

The authors declare no competing financial interests or personal relationships that could have influenced the work reported in this paper. The authors are not affiliated with Gurobi, IBM, Google, or any quantum hardware vendor.

Data Availability Statement

The complete QCCB benchmark (2,016 instances, all solver results, all quantum job data, evaluation scripts) is published at qccb-benchmark.org under CC BY 4.0.

Ethical Approval

This study is entirely computational. No human subjects, animal experiments, or patient data were involved. Ethical approval was not required.

ISSN: e-ISSN: 3117-6089 / p-ISSN: 3117-6070 Vol: 2024, Issues: 1, 2024

Appendix A

QCCB Benchmark Configuration and Problem Formulations

The following provides QCCB methodology implementation details and QUBO formulations for all six problem classes.

Benchmark Configuration and Classical Solvers

QUBO Formulations for Six Problem Classes